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Optimal monetary policy rules under persistent shocks

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Keywords

overlapping generations, random relocation model, monetary targeting, inflation targeting, real shocks, persistence

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Optimal Monetary Rules under Persistent Shocks

Joydeep Bhattacharya* and Rajesh Singh†

October 03, 2007

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The tug-o-war for supremacy between inflation targeting and monetary targeting is a classic yet timely topic in monetary economics. In this paper, we revisit this question within the context of a pure-exchange overlapping generations model of money where spatial separation and random relocation create an endogenous demand for money. We distinguish between shocks to real output and shocks to the real interest rate. Both shocks are assumed to follow AR(1) processes. Irrespective of the nature of shocks, the optimal inflation target is always positive. Under monetary targeting, shocks to endowment require negative money growth rates, while under shocks to real interest rates it may be either positive or negative depending on the elasticity of consumption substitution. Also, monetary targeting welfare-dominates inflation targeting but the gap between the two vanishes as the shock process approaches a random walk. In sharp contrast, for shocks to the real interest rate, we prove that monetary targeting and inflation targeting are welfare-equivalent only in the limit when the shocks become i.i.d.! The upshot is that persistence of the underlying fundamental uncertainty matters: depending on the nature of the shock, policy responses can either be more or less aggressive as persistence increases.

JEL Classification: E31, E42, E63

Keywords: real shocks, persistence, overlapping generations, random relocation model, monetary targeting, inflation targeting

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1 Introduction

The optimal conduct of monetary policy, whether to target the money growth rate or the inflation rate, has survived as one of the most contentious issues in monetary economics. Popular until recently, Milton Friedman’s (1960) “mechanical monetarism” advised central banks to stop setting interest rates and instead set the money growth rate permanently at the estimated growth rate of the real economy. Since the 80s, however, the dominant paradigm in the practice of monetary policy has shifted, bringing with it a renewed “dedication to price stability” via the *direct* control of inflation via interest rate targeting.

Poole (1970) presented the first formal treatment of the larger question: how should a monetary authority decide whether to use the money stock or the interest rate as the policy instrument. The debate at the time, as summarized by Poole, took the following shape: while some argued that “monetary policy should set the money stock while letting the interest rate fluctuate as it will”, others believed that monetary authorities should “push interest rates up in times of boom, and down in times of recession, while the money supply is allowed to fluctuate as it will.” Poole employed a stochastic IS-LM model and used volatility of aggregate output as the basis for comparison. He reached the conclusion that if “disturbances originated primarily in the IS function that summarized the real sectors of the economy [...], the money stock is the proper control instrument. But if the LM function, representing the monetary sector, is the source of the disturbances, the interest rate is the proper control variable” (Poole and Lieberman, 1972). The bottom-line advice was clear and extremely influential: when the shocks are real in nature, fix the money supply; if the shocks are monetary, fix the interest rate.

In this paper, we broaden the scope of the original ‘instrument problem’ within a modern optimizing framework and pose two questions of policy relevance.¹ First,

¹Our goal is not to “update” Poole’s results in a optimizing framework; rather to use his exercise as an inspiration to pose additional instrument problems.

when the shocks are “real”, does it matter whether they impinge on output or the real interest rate? Or more broadly, does it matter whether they are real income or real price shocks? Second, does the persistence of these shocks matter for the conduct of monetary policy? Our goal is strictly qualitative – to understand how optimal monetary policy (money growth and inflation rate targeting) should respond to real shocks and their persistence in a flexible-price “microfounded” model of money.²

To that end, we study a two-period lived two-island symmetric pure-exchange overlapping generations model in the tradition of Townsend (1987) where limited communication and stochastic relocation create an endogenous transactions role for fiat money. The competing asset to money is a commonly-available linear one-period storage technology. At the end of each period, a fraction of agents is relocated from one island to the other and they are allowed to bring only cash with them. This assumption solves the return-dominance problem of money. Risk averse agents seek insurance against this relocation risk. This justifies a role for generational banks that take deposits and holds a portfolio of cash reserves and storage on behalf of their clientele.

We study two variants of this model, one in which the young-age endowment and one where the return to the storage technology is exclusively random. We model these as AR(1) processes because our goal here is to study how changes in shocks’ persistence affect the optimal policy responses and the relative desirability of alternative instruments. For consistency, however, we ensure that the unconditional distributions of these processes remain invariant to such changes. This requires that the variance of innovations decline as the persistence of shocks increases.

Typically, with either uncertain endowment or real interest rate, banks will care not just about the current values of these variables but also their future realizations.

²Almost all the work done in this area has a quantitative focus and employ models with sticky or staggered prices, and very few, use welfare criteria – see Woodford (2003). Our approach is to ask: sans the added complexity of sticky prices, what can a reasonable neoclassical flexible-price model of money have to say about optimal monetary policy design?

For example, a bank this period cares about next period's endowment realization because the latter will potentially influence that period's money demand and hence the price level and therefore affect the return on money between this period and the next. But next period's money demand depends on the following period's endowment, and so on. We assume all agents know the relevant distributions of these shocks and form expectations about the returns on money and storage. If shocks are persistent, these expectations will be conditional on the current realizations. We focus solely on long run stationary equilibria under which agents expectations are coordinated across time, i.e., expectations of one generation are validated by the behavior of the next and so on ad infinitum.

In the case of endowment shocks, and with logarithmic (henceforth “log”) utility, we prove that irrespective of the degree of persistence, the optimal monetary target is a zero money growth rule while the optimal inflation rate target involves some inflation. We also show that monetary targeting welfare-dominates inflation-rate targeting but the gap between the two (i.e., the relative desirability of monetary targeting) reduces as the shocks become more persistent. In the limit, when the shocks to the endowment approach a random walk, the instrument problem vanishes: the two instruments become welfare-equivalent. We demonstrate numerically that similar results hold for more general CRRA preferences: as persistence rises, smaller policy responses in terms of money growth or inflation rate changes are needed, and the welfare gap between the two monetary regimes shrinks.

Results under real interest rate shocks are strikingly different. For log utility, irrespective of the persistence of shocks, both monetary and inflation targeting are *equivalent* and the optimal money growth rate/inflation rate in either case is zero. For more general CRRA utility, *in sharp contrast* to the result under endowment shocks, monetary targeting and inflation targeting are welfare-equivalent only in the limit when the shocks become i.i.d.! The magnitude of the policy response under either regime (money growth rate or inflation rate targeting) as well as the welfare

gap between the two increases with the persistence of these shocks. The elasticity of intertemporal substitution of consumption plays a critical role here. As in the case of endowment shocks, an optimal inflation rate under real interest shocks is always non-negative, but optimal money growth rate is negative only when consumption is elastic (relative to log); else, it is positive. More interestingly, inflation targeting dominates monetary targeting when consumption is elastic (relative to log); otherwise, the latter performs better.

The upshot is that even in flexible-price environments, the nature of real shocks – whether they impinge on output or the interest rate – *and* their persistence, matter crucially for optimal monetary policy design. An information about the nature of the shock alone is no longer sufficient.

The random relocation with limited communication model was popularized by Champ, Smith, and Williamson (1997) and has been used to investigate myriad monetary policy issues in Schreft and Smith (1997), Smith (2002), Antinolfi, Huybens, and Keister (2001), Antinolfi and Keister (2006), Gomis-Porqueras and Smith (2003), Bhattacharya, Haslag, and Russell (2005), Haslag and Martin (2007), among many others. The random relocation model is attractive because it includes a genuine reason why money is held even when dominated in return; it also allows for easy inclusion of institutions such as Diamond-Dybvig (1983) style banks. Another strong appeal of this model is its analytical tractability. To date, however, researchers have either worked with deterministic versions of the model – such as Gomis-Porqueras and Smith (2003)– or at best, with i.i.d. shocks, as in Bhattacharya and Singh (2007).³ Independent of its aforesaid contribution to the area of optimal monetary policy design, our paper makes strides in the analytical treatment of persistent shocks in this popular model of money.

A paper close in spirit to ours is Kryvtsov et. al. (2007). In an overlapping gener-

³The current paper is in some respects a companion piece to Bhattacharya and Singh (2007). However, allowing for persistent AR(1) shocks and adding shocks to the real interest rate makes the current analysis considerably more general and formidable.

ations economy similar in some respects to ours, they show that increased persistence of endowment shocks requires a smaller inflationary intervention. This is primarily because with higher persistence, the transitory component of the shock becomes less important.⁴ Although our policy results for the case of endowment shocks have a similar flavor, the underlying reasons are somewhat different. In our set up *sans* shocks, as is well known, a fixed money supply (or equivalently, a constant price of consumption) is the optimal policy.⁵ It is only when the shocks induce banks to deviate from the steady state allocations that a need for optimal policy to deviate from the fixed money supply rule arises. In particular, the higher the impact of shocks, the higher will be the required deviation. A higher persistence essentially reduces the (conditional) uncertainty of next period's endowment and therefore price level uncertainty is reduced. Consequently, irrespective of the monetary regime, the policy response becomes *less* aggressive. In contrast, when the shocks impinge on the real interest rate, we find that the policy response can become *more* aggressive with increased persistence. Here, if the returns are i.i.d., current realizations (on investments made last period) are irrelevant for the investment choices of the current generation. Instead, if these returns are persistent, their expectational relevance for the next period's return makes current investments contingent on current realizations. As a result, investments into cash fluctuate over time. Indeed, the higher the persistence, the more volatile are the cash balances. Consequently, policy responses strengthen

⁴In their model, agents "precautionarily" carry more money than what is optimal from a planner's perspective. Therefore, to discourage such savings, the optimal inflation rate is positive. With i.i.d. shocks, all changes are temporary and the precautionary motive is strong. The higher the current endowment, the stronger is the precautionary motive, and the higher is the optimal inflation rate. With persistence, a change in endowment has a permanent component that does not require a "proportional" precautionary response. The policy need not discourage savings as aggressively. As a result, the volatility of optimal inflation declines with the persistence of shocks.

⁵In equilibrium, relocated old agents use money to purchase out of the endowment deposited by the current young. From a planner's perspective, ex-ante, what is consumed by the relocated agents can not be stored. The social (planner's) opportunity cost of movers' consumption relative to that of non-movers is thus the lost return on storage. In a decentralized economy, this is implemented by keeping the money supply fixed, i.e., a zero *net* return on money. See Bhattacharya and Singh (forthcoming) for a detailed discussion.

with shock persistence.

Gomis-Porqueras and Smith (2003) also study output and real interest rate fluctuations; specifically, they consider two-period *deterministic* cycles of output, real interest rate, and relocation shocks. They show that real interest rate shocks require relatively higher nominal interest rate smoothing than do endowment shocks. The result relies critically on their assumption of elastic (relative to log) utility. We show that the ranking of interest rate targeting vis-à-vis monetary targeting crucially depends on the elasticity of intertemporal substitution. Moreover, not only do we evaluate the best rule within each monetary regime, we also explicitly rank the two regimes, something that they do not.

The plan for the rest of the paper is as follows. In the next section, we outline the baseline model and characterize decentralized allocations. In Section 3, we study the role of endowment uncertainty in shaping the optimal choice of monetary instruments. In Section 4, we do the same with real interest rate shocks. Both these sections also include the results from the computational experiments under CRRA utility. Section 5 concludes. Proofs of all major results are in the appendices.

2 The Model

2.1 Preliminaries

We present a model economy that is populated by a unit mass of two-period overlapping generations of agents located in two spatially separated locations.⁶ Time is denoted by $t = 1, 2, \dots, \infty$. Each two-period-lived agent is endowed with $w_t > 0$ units of this good at date t when young and nothing when old. We assume w is stochastic and that w is revealed at the start of any date; the specifics are provided below in Section 3.

⁶Our formulation follows Townsend (1987), and Schreft and Smith (1997). Only a concise description of the environment is provided here; the reader is referred to Schreft and Smith (1997) for details.

Only old-age consumption is valued. Let c_t denote old-age consumption of the members of the generation born at date t ; their lifetime utility is given by $u(c) = [c^{1-\phi} - 1] / (1 - \phi)$, with $\phi > 0$, and $u(c) = \ln c$ when $\phi = 1$.

All agents have access to two assets, storage and money. Each unit of the consumption good put into storage at date t yields x_{t+1} units of the consumption good at date $t + 1$. We assume x is stochastic and that it is revealed at the start of any date; the specifics are provided below in Section 4. The other asset is fiat currency (money) that agents may acquire through sale of their endowment. Let p_t denote the price level at date t . Then the gross real rate of return on money (R_{mt}) between period t and $t + 1$ is given by $R_{mt} = p_t / p_{t+1} = \frac{1}{\pi_t}$, where π_t denotes the inflation rate between period t and $t + 1$. We will assume that money is dominated in its return (see below).

The sequence of events is as follows. Agents receive their endowment at the start of a period. Towards the end of the period, after the savings decisions have been made, a fraction α of randomly chosen agents from each location is relocated to the other location. A relocated agent cannot collect the return on any goods she has stored, or that have been stored on her behalf, since goods cannot be transported across locations. However, if an agent is carrying fiat currency when she is relocated, then the currency is relocated with it.

Young agents may either invest their endowment directly into one or both the assets or go through a bank that takes deposits and invests in these assets; the specifics of the banks' problem will follow later. Under standard assumptions discussed in Schreft and Smith (1997), agents will find it in their best interest to deposit their entire endowment into a bank before they learn their relocation status. The bank pools the goods deposited by all the young agents and uses them to acquire a portfolio of stored goods and fiat currency. It issues claims to agents whose nature, timing, and size are contingent on their relocation status. If an agent does not get relocated (henceforth, a non-mover), she gets a return on her deposit next period funded by the

goods the bank has stored. If she gets relocated (henceforth, a mover), then she gets a return on her deposit in the same period in the form of a fiat currency payment (whose real value will depend on the following period's price level) funded by the bank's holdings of fiat currency.

The quantity of money in circulation at the end of period $t \geq 1$, per young agent, is denoted M_t . Let $m_t \equiv M_t/p_t$ denote real money balances at date t . The government conducts monetary policy in one of two possible ways. The first, called “monetary targeting”, is one where the government changes the nominal stock of fiat currency at a fixed non-stochastic gross rate $\mu > 0$ per period, so that $M_t = \mu M_{t-1}$ for all t . The second, called “inflation targeting”, is one where the government changes the nominal stock of fiat currency in such a way as to keep the long-run gross inflation rate fixed at π . All money injections are implemented through lump sum transfers (τ) to the young agents. The period t budget constraint of the government is

$$\tau_t = \frac{M_t - M_{t-1}}{p_t} = m_t - m_{t-1}R_{m,t-1} \quad (1)$$

for all $t \geq 1$.

2.2 The bank's problem

The asset holdings of young agents are assumed to be costlessly intermediated by perfectly competitive banks. Under this assumption, banks' equilibrium profits from any generation is zero. One can equivalently think of banks being created each period by the current cohort of agents who wish to maximize the ex-ante overall return on their cash-storage portfolio while keeping “sufficient” cash for the movers.

Specifically, every young agent deposits her after-tax/transfer income in the bank. The banks divide their deposits between stored goods s_t and real balances of fiat currency m_t , so that

$$w_t + \tau_t = m_t + s_t. \quad (2)$$

At the time the bank solves its problem, the current endowment w_t and the return x_t on goods stored during previous period is known; so is the equilibrium price level p_t . But w_{t+1} , x_{t+1} , and p_{t+1} has not been realized yet. The bank cares about x_{t+1} as it directly affects the consumption of non-movers. Additionally, as x_{t+1} helps predict x_{t+2} , it may also influence the next period's money demand and the price level, which in turn affects the rate of return on money between this period and the next. The indirect effect also holds for w_{t+1} through its effect on the next period's money demand.

Evidently the bank cannot promise a fixed real return to its depositors. All it can do is let the depositors know the amount of nominal balances being kept aside for their use on the other island. The bank knows the stochastic process for w_{t+1} and x_{t+1} and forms expectations on the return on money R_{mt} ; in a rational expectations equilibrium, these expectations are correct. We focus solely on long-run stationary equilibria under which expectations are coordinated across time, i.e., expectations of one generation are validated by the behavior of the next and so on ad infinitum.

Define $\gamma_t \equiv \frac{m_t}{w_t + \tau_t}$ as the ratio of cash reserves to deposits. Below, when we specify shock processes, we also provide conditions to ensure that money is dominated in return by storage. Then the bank will never want to carry cash balances across time. In that case, the bank's problem is given by

$$\max_{\gamma_t \in [0,1]} \alpha E_t \left(u \left(\frac{\gamma_t}{\alpha} R_{mt} (w_t + \tau_t) \right) \right) + (1 - \alpha) E_t \left(u \left(\frac{(1 - \gamma_t)x_{t+1}}{(1 - \alpha)} (w_t + \tau_t) \right) \right), \quad (3)$$

where E_t is an expectations operator *conditional* on date t information. Let $c_{mt} \equiv \frac{\gamma_t}{\alpha} (w_t + \tau_t)$ and $c_{nt} \equiv \frac{1 - \gamma_t}{1 - \alpha} (w_t + \tau_t)$, the consumption allocations offered by the bank to its moving and non-moving clientele. The first order condition for this problem is given by

$$E_t (u' (c_{mt}) \cdot R_{mt}) = E_t (x_{t+1} \cdot u' (c_{nt})). \quad (4)$$

Eq. (4) equates the expected marginal value of a unit of endowment saved as cash with its expected marginal value were it instead kept in storage.

Under CRRA preferences, i.e., $u(c) = \frac{c^{1-\phi}-1}{1-\phi}$ with $\phi > 0$, the first order condition to the bank's problem reduces to

$$\alpha^\phi \gamma_t^{-\phi} E_t \left(R_{mt}^{1-\phi} \right) = (1-\alpha)^\phi [(1-\gamma_t)]^{-\phi} E_t \left(x_{t+1}^{1-\phi} \right), \quad (5)$$

which further simplifies to

$$\gamma_t = \frac{\alpha}{\alpha + (1-\alpha) \left(\frac{E_t(R_{mt}^{1-\phi})}{E_t(x_{t+1}^{1-\phi})} \right)^{-\frac{1}{\phi}}}. \quad (6)$$

Monetary policy influences the optimal γ since it determines the relative return on money (R_m).

It is possible to make further analytical progress under the assumption of logarithmic utility. The bank's problem is now rewritten as

$$\max_{\gamma_t \in [0,1]} \left\{ \alpha E_t (\ln R_{mt}) + \alpha \ln \left(\frac{\gamma_t}{\alpha} \right) + (1-\alpha) \ln \left(\frac{1-\gamma_t}{1-\alpha} \right) + (1-\alpha) E_t (x_{t+1}) + \ln (w_t + \tau_t) \right\}. \quad (7)$$

Observe that the bank knows the current period endowment and takes the return on money and the size of the transfer as given. Therefore, the bank's choice of γ will respond only to the second and the third terms of the previous expression. Then, the choice of γ_t is given by

$$\gamma_t = \alpha \text{ for all } t, \quad (8)$$

which, of course, may also be obtained from (6) by substituting $\phi = 1$. As is well known, with logarithmic utility, banks allocate deposits across the two assets to provide consumption to the two types in proportion to their population shares. As a result, the choice of γ is not state-contingent. This will not be the case in the more general CRRA formulation, as is evident from (6).

2.3 Equilibrium under monetary and inflation targeting

Enroute to solving for optimal monetary rules, we describe the monetary equilibrium under the two targeting rules. Under **monetary targeting**, the government fixes the

money growth rate at μ . Since $\tau_t = \frac{M_t - M_{t-1}}{p_t} = m_t \left(1 - \frac{1}{\mu}\right)$ and $m_t = \gamma_t (w_t + \tau_t)$, we have

$$m_t = \frac{\gamma_t w_t}{1 - \gamma_t \left(1 - \frac{1}{\mu}\right)}; \quad w_t + \tau_t = \frac{w_t}{1 - \gamma_t \left(1 - \frac{1}{\mu}\right)}. \quad (9)$$

Hence, the equilibrium return on money is given by

$$R_{mt} = \frac{m_{t+1}}{\mu m_t} = \frac{1}{\mu} \frac{\gamma_{t+1} w_{t+1}}{\gamma_t w_t} \frac{1 - \gamma_t \left(1 - \frac{1}{\mu}\right)}{1 - \gamma_{t+1} \left(1 - \frac{1}{\mu}\right)}. \quad (10)$$

With logarithmic utility, $\gamma_t = \alpha$ for all t (see (8)), and the above reduces to $R_{mt} = \frac{1}{\mu} \frac{w_{t+1}}{w_t}$.

Given the exogenous process for $\{x, w\}$ and money growth rate μ , a *stationary equilibrium* of this economy consists of a time-invariant portfolio allocation function $\gamma(w_t, x_t)$ and a return function R_{mt} (given by (9) and (10)) such that given R_{mt} , $\gamma(w_t, x_t)$ solves the banks' problem (3) and

$$R_{mt} = \frac{1}{\mu} \frac{\gamma(x_{t+1}, w_{t+1}) w_{t+1}}{\gamma(x_t, w_t) w_t} \frac{1 - \gamma(x_t, w_t) \left(1 - \frac{1}{\mu}\right)}{1 - \gamma(x_{t+1}, w_{t+1}) \left(1 - \frac{1}{\mu}\right)}. \quad (11)$$

Under **inflation targeting**, the government fixes the inflation rate at π . The real return to money is

$$R_{mt} = \frac{1}{\pi}. \quad (12)$$

Here, there is no uncertainty about the rate of return on money. The government conducts monetary policy via time-varying taxes and transfers to ensure money is an asset with a fixed real return. Since $\tau_t = \frac{M_t - M_{t-1}}{p_t} = m_t - \frac{m_{t-1}}{\pi}$, we get $m_t = \gamma_t (w_t + \tau_t) = \gamma_t \left(w_t + m_t - \frac{m_{t-1}}{\pi}\right)$ implying

$$m_t = -\frac{\gamma_t}{(1 - \gamma_t)\pi} m_{t-1} + \frac{\gamma_t}{1 - \gamma_t} w_t \quad (13)$$

Thus, real balances follow an AR(1) process under inflation targeting. For future reference, under log utility with $\gamma_t = \alpha$, (13) reduces to

$$m_t = -\rho_m m_{t-1} + \frac{\alpha}{1 - \alpha} w_t \quad (14)$$

where $\rho_m \equiv \frac{1}{\pi} \frac{\alpha}{1-\alpha}$ is the persistence term.

A stationary equilibrium of this economy is described by a time-invariant portfolio allocation function $\gamma(\pi)$, that given π and the stochastic process for x solves banks' problem described by equation (3) – given by (6). The solution leads to a stationary process for real money demand governed by (13).

As discussed in Bhattacharya and Singh (forthcoming), in steady states, a planner constrained by limited communication faces a return of x on stored goods; such a planner who allocates w between the movers and the non-movers would choose an allocation (c_m, c_n) so as to maximize $\alpha u(c_m) + (1 - \alpha) u(c_n)$ subject to $\alpha c_m + (1 - \alpha) c_n / x = w$. The marginal condition is

$$u'(c_m) = x \cdot u'(c_n).$$

This is the “intratemporal efficiency” or “intragenerational efficiency” condition connecting marginal utilities of movers and non-movers at any date. In a steady state, a government trying to replicate the planning solution would face a static problem and hence would need to pay attention solely to this intratemporal margin. With shocks, however, the government’s problem does not remain static. An intertemporal (intergenerational) margin appears because shocks hit different generations asymmetrically. Now the government pays attention to providing some amount of intergenerational insurance. To achieve this, the government may opt to trade off intratemporal for intertemporal efficiency and this causes optimal monetary policy to deviate from whatever policy achieves intratemporal efficiency alone.

A road-map of what lies ahead is in order. We start by analyzing the optimal monetary rules under monetary and inflation targeting policies in the case where endowments follow an AR(1) process but storage return is fixed. We are able to derive clean analytical results for the case of logarithmic utility. We go on to compare stationary welfare across the two targeting policies. We then repeat these exercises in the case where the endowment is fixed but the return to storage follows an AR(1) process. In both cases, we extend the scope of our results to the case of CRRA utility

by means of numerical computations.

3 Endowment uncertainty

We start off by fixing the return on storage to x for all dates and allowing the endowment process to be stochastic. Shocks to the endowment are intended to represent real output shocks. Our goal is to investigate how optimal monetary policy responds to such shocks, and in particular, to the persistence of such shocks.

We assume that w_t follows an AR(1) process of the form:

$$w_t = \rho_w w_{t-1} + \varepsilon_t, \quad (15)$$

where ε is i.i.d. with mean $(1 - \rho_w) \bar{w}$ and variance $(1 - \rho_w^2) \sigma_w^2$, and where \bar{w} and σ_w^2 are the unconditional mean and variance of the endowment process. This specification allows us to study how optimal policies vary with the persistence (ρ_w) of the shocks while keeping the unconditional mean and variance of these shocks constant. We also assume that $x > \frac{\rho_w}{\mu} + (1 - \rho_w) \frac{w^e}{w_{\min}}$. This ensures that $x > E_t(R_{mt})$ always holds, and banks never carry cash for the non-movers.

We adopt an ex-ante measure of welfare. This allows us to obtain the stochastic analog of the golden rule in a stochastic overlapping generations economy with finitely lived agents. Here, a generation's welfare is defined as its lifetime expected utility where an unconditional expectation is taken with respect to stationary distributions of exogenous as well as endogenous variables. This measure, by construction, treats all generations symmetrically and makes each of them "representative". The unconditional expectation ensures that the derived policy rules are state-uncontingent or "timeless".

We first consider the optimal money growth rate under monetary targeting, then characterize the optimal inflation rate under inflation targeting, and finally establish a ranking between the two.

3.1 Monetary targeting

Monetary policy has different effects on the two groups, movers and non-movers. The latter's consumption is given by $x(w_t + \tau_t)$ while the formers' by $R_{mt}(w_t + \tau_t)$. Using (9), it is easily verified that the consumption of non-movers is given by $c_{nt} = \frac{xw_t}{1-\alpha+\frac{\alpha}{\mu}}$ and that of the movers by $c_{mt} = \frac{1}{\mu} \frac{w_{t+1}}{1-\alpha+\frac{\alpha}{\mu}}$. Then, indirect utility as a function of μ at t is obtained by evaluating (7) at $\gamma_t = \alpha$ and using the equilibrium return given by (10):

$$W_t(\mu) = \alpha E_t \{\ln w_{t+1} | w_t\} + (1 - \alpha) \ln(xw_t) - \ln \left[1 - \alpha \left(1 - \frac{1}{\mu} \right) \right] - \alpha \ln \mu. \quad (16)$$

Notice that μ has no effect on the first two terms on the r.h.s. of the welfare expression. Ex-ante stationary aggregate welfare is defined as $W^\mu \equiv E \{W(\mu)\}$ where E is the unconditional expectation. Thus, we have

$$W^\mu = (1 - \alpha) \ln x + E(\ln w) - \ln \left[1 - \alpha \left(1 - \frac{1}{\mu} \right) \right] - \alpha \ln \mu \quad (17)$$

What μ is the best from the standpoint of stationary welfare? We define $\tilde{\mu} \equiv \arg \max \{W^\mu\}$. Since W^μ is assumed to be concave in μ , $\tilde{\mu}$ solves $\frac{d}{d\mu} W^\mu = 0$; [notice $\tilde{\mu}$ maximizes the last two terms in (17)]. For future reference, we define $\tilde{W}^\mu \equiv \max_{\mu} W^\mu$.

Proposition 1 *Under logarithmic utility, irrespective of ρ_w , the optimal monetary policy is to keep the money supply fixed, i.e., $\tilde{\mu} = 1$.*

Notice from equation (16) that the terms containing μ are independent of both the current and/or future endowment. In that case, optimal monetary policy can safely ignore the aforesaid intergenerational margin and work solely to reach an efficient intragenerational margin.

3.2 Inflation targeting

The stochastic process for real balances under inflation targeting as given by (14) can be rewritten as

$$m_t = -\rho_m m_{t-1} + \frac{\gamma}{1-\gamma} w_t = \frac{\alpha}{1-\alpha} \sum_{s=0}^{\infty} (-\rho_m)^s w_{t-s}.$$

This, when combined with the AR(1) process for endowments, yields

$$m_t = \frac{\alpha}{1-\alpha} \left[\sum_{s=0}^{\infty} \rho_w^s \varepsilon_{t-s} - \rho_m \sum_{s=1}^{\infty} \rho_w^{s-1} \varepsilon_{t-s} + \rho_m^2 \sum_{s=2}^{\infty} \rho_w^{s-2} \varepsilon_{t-s} - \rho_m^3 \sum_{s=3}^{\infty} \rho_w^{s-3} \varepsilon_{t-s} + \dots \right].$$

Denote the mean and variance of the stochastic process for m defined in (14) by \bar{m} and σ_m^2 respectively. The following lemma contains some pertinent information about the mean and variance of the stochastic process for m .

Lemma 1 *For log utility, under inflation targeting, the mean and the variance of the stochastic process for real balances are given by:*

$$\begin{aligned} \bar{m} &= \frac{\alpha}{1-\alpha} \frac{\bar{w}}{1+\rho_m}; \\ \sigma_m^2 &= \frac{\alpha^2}{(1-\alpha)^2} \sigma_w^2 (1-\rho_w^2) \left(1 + \frac{1}{(\rho_w + \rho_m)^2} \left[\frac{\rho_w^4}{1-\rho_w^2} - 2 \frac{\rho_m^2 \rho_w^2}{1+\rho_w \rho_m} + \frac{\rho_m^4}{1-\rho_m^2} \right] \right) \end{aligned} \quad (18)$$

Moreover, the variance declines monotonically with ρ_w .

Since the unconditional mean and variance, \bar{w} and σ_w^2 , are fixed, Lemma 1 shows that the variance of real balances or equivalently net-of-transfer income (recall that $w + \tau = m/\alpha$) depends on the persistence of the endowment process *and* the persistence of monetary process. Notice however, that mean real balances depends solely on ρ_m and not on ρ_w .

The following lemma, which compares $\frac{\sigma_w}{\bar{w}}$ with $\frac{\sigma_m}{\bar{m}}$, will permit further progress towards evaluation of the optimal inflation target and comparison across the two targeting schemes.

Lemma 2 *As ρ_w increases $\frac{\sigma_m}{\bar{m}} / \frac{\sigma_w}{\bar{w}}$ declines. In particular,*

$$\left. \begin{aligned} \frac{\sigma_m^2}{(\bar{m}^e)^2} &= \frac{\frac{((1-\alpha)\pi+\alpha)^2}{(1-\alpha)^2\pi^2-\alpha^2}}{\frac{\sigma_w^2}{(\bar{w}^e)^2}} > 1 \text{ for } \rho_w = 0 \\ &> 1 \text{ for all } \rho_w = \rho_m \in (0, 1) \\ &1 \text{ as } \rho_w \rightarrow 1 \end{aligned} \right\}$$

Thus, when endowment shocks are highly persistent, the per-unit variance of net-of-transfer income approaches that of the endowment process.

Under inflation targeting, there is no uncertainty regarding the return on money. The only remaining uncertainty is about the net-of-transfer income which is the sum of the endowment and the transfer income. When the persistence of the endowment process increases, it gains relatively higher importance (relative to the stochastic process for money balances which determines the transfer income) in the determination of the net-of-transfer income. In the limit, as the endowment process becomes a random walk, the significance of monetary process vanishes completely. As the unconditional variance of the endowment process is constant, a reduction in the relative importance of monetary process reduces the unconditional variance of the net-of-income process. An increase in the persistence of endowment process thus makes inflation targeting more desirable, as will become clear further below.

Evaluating (7) at $\gamma_t = \alpha$ and using (12) obtains indirect utility at date t as

$$W_t(\pi) = -\alpha \ln \pi + (1 - \alpha) \ln x + \ln(w_t + \tau_t) = -\alpha \ln \pi + (1 - \alpha) \ln x + \ln\left(\frac{m_t}{\alpha}\right).$$

It is apparent that π has two effects on welfare, one through its effect on the return on money (captured by the $-\alpha \ln \pi$ term above) and the other via its effect on post-tax/transfer income (captured by the $\ln(w_t + \tau_t)$ or $\ln\left(\frac{m_t}{\alpha}\right)$ terms above).

Having characterized the first and second moment properties of the net-of-transfer income under inflation targeting, we are now ready to study the properties of an optimal inflation target. To do so, we first define the ex ante stationary aggregate welfare as

$$W^\pi \equiv E(W(\pi)) = -\alpha \ln \pi - \ln \alpha + (1 - \alpha) \ln x + E(\ln m). \quad (19)$$

Notice that the last term on the r.h.s. of (19) corresponds to $E(\ln(w_t + \tau_t))$ or the mean value of log post tax income. We define the optimal inflation rate as $\tilde{\pi} \equiv \arg \max \{W^\pi\}$ and its corresponding welfare as $\tilde{W}^\pi \equiv \max_\pi \{W^\pi\}$. Henceforth, we *assume* that W^π is strictly concave in π ; then $\tilde{\pi}$ solves $\frac{d}{d\pi} W^\pi = 0$.

Proposition 2 *Under logarithmic utility, optimal inflation targeting involves setting a positive inflation rate, or $\tilde{\pi} > 1$ for all $\rho_w \in [0, 1)$. Furthermore, $\lim_{\rho_w \rightarrow 1} \tilde{\pi} = 1$.*

The proof of Proposition 2 relies on a second-order Taylor approximation of the last term on the r.h.s. of (19) around \bar{m} .⁷ An intuition for this result is as follows. The first thing to note is that while inflation targeting eliminates rate of return uncertainty, the uncertainty about the post-tax/transfer income ($w + \tau = \frac{m}{\alpha}$) remains. A risk-averse agent would thus prefer to have the highest expected value for m with minimum accompanying volatility. It can be checked from Lemmas 1 and 2 that raising inflation rate achieves both objectives, it follows that choosing a positive inflation rate ($\pi > 1$) may be desirable.

Intuitively, in order to give $\frac{m_t}{\pi}$ to date t movers the government will have to tax the date $t + 1$ young such that their storage allocation, $(1 - \alpha)(w_{t+1} + \tau_{t+1})$, leaves $\frac{m_t}{\pi}$ of goods for date t mover's consumption. In other words, taxes/transfers should be such that $(1 - \alpha)(w_{t+1} + \tau_{t+1}) = w_{t+1} - \frac{m_t}{\pi}$. This is the rationale behind (14), which is rewritten below as

$$(w_{t+1} + \tau_{t+1}) = -\frac{\alpha}{(1 - \alpha)\pi} (w_t + \tau_t) + \frac{1}{1 - \alpha} w_{t+1}. \quad (20)$$

Equation (20) makes clear that the autocorrelation between total income at two adjacent periods is negative and the *strength* of this correlation becomes smaller as π rises. Thus setting $\pi > 1$ may improve intertemporal efficiency; this way shocks to income do not get transmitted over time as easily.

Proposition 2 also states that as the persistence of endowment shock increases, the optimal inflation target approaches unity. This directly follows the results stated

⁷Observe that $E \ln m = \ln \bar{m} - \frac{1}{2} \frac{\sigma_m^2}{\bar{m}^2}$ and this is where Lemma 2 helps.

in Lemma 2. As the persistence of endowment process gets larger the per unit net of transfer income volatility under inflation targeting approaches that of the endowment process. As a result, the policy correction through inflation needs to be less aggressive.

We now proceed to answer the question: which targeting regime, set at its own optimal rate, achieves higher welfare?

Proposition 3 *Under logarithmic utility, optimal targeting of the money growth rate is stationary-welfare superior to optimal targeting of the inflation rate for all $\rho_w < 1$. The welfare gap between the two regimes shrinks as ρ_w increases; as $\rho_w \rightarrow 1$, both regimes are stationary-welfare equivalent.*

Recall that optimal monetary targeting involves setting $\mu = 1$ (fixing the money supply) thereby making the post-tax/transfer income exactly equal to the endowment. In this setting, as discussed earlier, both non-movers' and movers' consumption variability is solely due to the endowment uncertainty. On the other hand, optimal inflation targeting involves fixing the inflation rate thereby eliminating any uncertainty with respect to the return on money; the residual uncertainty, in this case, is with regard to the post-tax/transfer income.

Why is monetary targeting superior? A fixed money supply rule achieves ex-ante intratemporal efficiency; it does/can not affect the fundamental endowment uncertainty, captured by σ_w^2 . Compare this to a zero net inflation rate policy. Of course, as we have seen, $\pi = 1$ can achieve ex-ante intratemporal efficiency; however, it is associated with a higher volatility of post-tax income. In order to reduce the per unit volatility of net-of-transfer income ($\frac{\sigma_m}{\bar{m}}$), π needs to be increased. The optimal π thus trades off intratemporal efficiency against the benefit received from the reduction in income volatility. Yet, as Lemma 2 shows, for any degree of persistence of endowment shocks, the per-unit volatility of net-of-transfer income under any inflation rate is *higher* than the endowment volatility, which equals the net-of-transfer income volatility under monetary targeting. Overall, relative to monetary targeting, optimal

inflation targeting distorts the intratemporal efficiency margin and additionally leaves the net-of-transfer income more volatile. This makes it less desirable overall.

Proposition 3 also states that as the persistence of endowment process increases, the welfare under inflation targeting approaches that achievable under monetary targeting. Once again, the result follows from Lemma 2. As ρ_w increases, the volatility of net-of-transfer income under inflation targeting decreases and the welfare gap accordingly shrinks. When the endowment process is a random walk, the significance of the monetary process in net-of-transfer income determination completely vanishes, and net-of-transfer income volatility not only equals that of endowment volatility, but is also independent of the inflation rate. Since intratemporal efficiency is ensured, the inflation rate instrument can play no additional role. As under monetary targeting, it is optimal to have $\tilde{\pi} = 1$; hence, an identical welfare is obtained.

3.3 CRRA utility

We now extend our analysis to incorporate the more general CRRA utility form: $u(c) = [c^{1-\phi} - 1] / (1 - \phi)$ where ϕ is the coefficient of relative risk aversion. Our objective here is to verify whether the flavor of the results from Section 3 continue to obtain for ϕ away from unity. Since it is not possible to pursue this analytically, we will resort to numerical analysis below.

By combining (5) with (3), the expression for period t welfare is given by

$$W_t \equiv \frac{(1 - \alpha)^\phi (w_t + \tau_t)^{1-\phi} (1 - \gamma_t)^{-\phi} E_t \left(x_{t+1}^{1-\phi} \right) - 1}{1 - \phi} = \left(\frac{\alpha}{\gamma_t} \right)^\phi \frac{(w_t + \tau_t)^{1-\phi} E_t (R_{mt}) - 1}{1 - \phi}. \quad (21)$$

Under monetary targeting $\gamma_t(w_t)$, $w_t + \tau_t$, and R_{mt} are given by (6), (9), and (10), respectively. Thus, for given probability distributions for α and w , the equilibrium function γ is obtained as a fixed point of (6). Evidently, under monetary targeting, the equilibrium γ_t [denoted $\gamma_t(\mu)$] is a function of μ , and the period t realization

of w . Under inflation targeting $R_m = \pi^{-1}$. Then γ is obtained from (6), and then $w_t + \tau_t = \frac{m_t}{\gamma}$ is obtained from (13).

Next, the optimal policies and optimal welfare levels are defined as

$$\tilde{W}^i \equiv \max_i \{W^i\}, \tilde{i} \equiv \arg \max \{W^i\}, \quad i \in \{\pi, \mu\}$$

Finally, we represent \tilde{W}^i in terms of its consumption equivalent \tilde{c}^i by using

$$\tilde{W}^i = \left[(\tilde{c}^i)^{1-\phi} - 1 \right] / (1 - \phi).$$

Our choice of parametric specification is as follows. We fix $\alpha = 0.2$ and assume that the long-run distribution of w is log-normal. For the AR(1) specification, we assume

$$\ln w_t = \rho_w \ln w_{t-1} + \varepsilon_t,$$

where $\varepsilon \sim N(\mu(1 - \rho_w), \sigma^2(1 - \rho_w^2))$; $\rho_w = 0$ obviously nests the i.i.d. specification. Below we present results for $\rho_w = 0, 0.5$, and 0.9 . In particular, we compare optimal money growth and inflation rates under the two policies, along with their respective welfare levels, for $\phi \in [0.5, 2.1]$.⁸

A few words about the computational algorithm is in order. Under monetary targeting, the main step entails computing the fixed point of γ as a function of w , depending on the nature of the shock. To do so, we guess an initial function,⁹ and numerically iterate on (6) to convergence. This is done for a fixed μ . Once the γ function is obtained, evaluating (21) by averaging over a large number of simulations obtains the ex-ante welfare. By repeating this exercise for different values of μ , we easily obtain $\tilde{\mu}$ and \tilde{W}^μ . Under inflation targeting, we first fix π . This yields γ directly. After assuming an initial value of $m_0 = m^e$, we let the computer simulate (13), and for each observation of α or w , and m compute (21). An average obtained

⁸We find that $\phi \in [0.5, 2.1]$ is a fairly representative range, and the qualitative nature of our results continue to hold when this range is enlarged.

⁹Our initial guess is $\gamma(w_t) = \alpha$. The convergence to the equilibrium function at any desired accuracy is reasonably fast.

from the previous step yields ex-ante welfare. By repeating this exercise for different values of π , we obtain $\tilde{\pi}$ and \tilde{W}^π .

We have analytically established that when $\phi = 1$ (log utility), $\tilde{\mu} = 1$ and $\tilde{\pi} > 1$.

Figure 1(a) shows that $\tilde{\pi} > 1$ for all ϕ , and Figure 1(b) shows that $\tilde{\mu} \leq 1$ for all ϕ .¹⁰

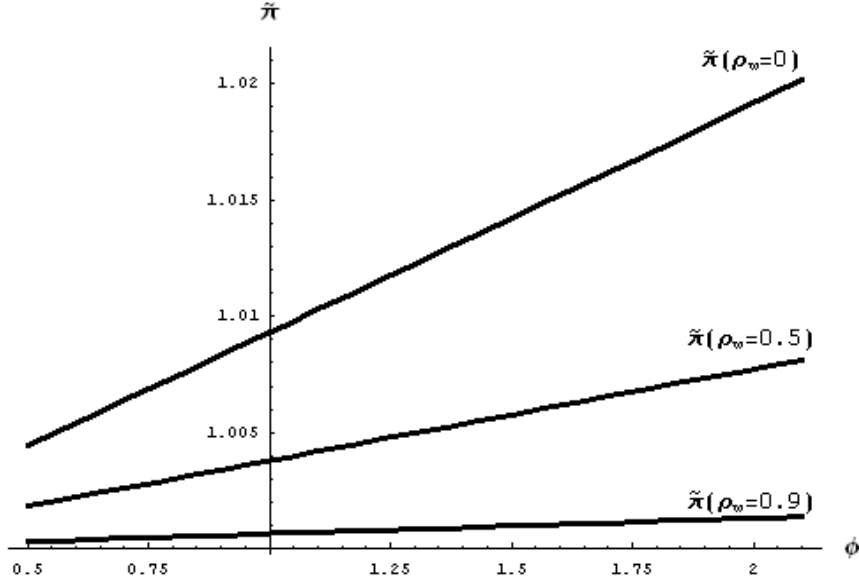


Figure 1(a): Optimal π against ϕ (endowment shocks)

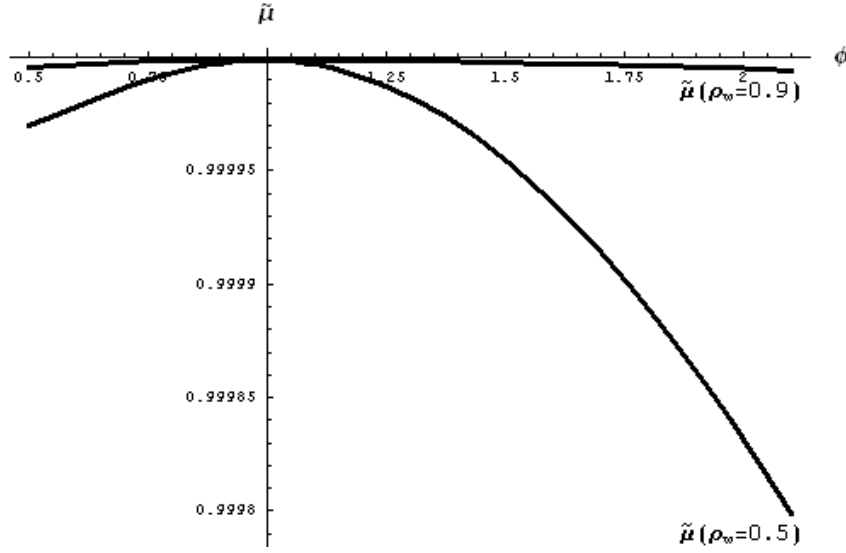


Figure 1(b): Optimal μ against ϕ (endowment shocks)

¹⁰The $\tilde{\mu}$ curve for $\rho_w = 0$ – the i.i.d. case – follows the same shape as the other two, is tangential to them at $\phi = 1$, and lies strictly below $\rho_w = 0.5$ at all other ϕ . If presented together, $\tilde{\mu}$ for $\rho_w = 0.9$ becomes indistinguishable from the x -axis.

As under log utility, $\tilde{\pi} > 1$ reduces the net-of-transfer income volatility. Figure 1(a) shows that for any given ρ_w , $\tilde{\pi}$ is increasing in ϕ . Intuitively, as ϕ gets higher i.e., the utility is more concave, the income volatility hurts more. A more aggressive policy response is needed and thus $\tilde{\pi}$ is increasing in ϕ . Figure 1(a) also shows that, for any given ϕ , $\tilde{\pi}$ is decreasing in ρ_w . Recall from the discussion following Lemma 2 that net-of-transfers income volatility is decreasing in the persistence of the endowment shocks. As a result, the policy is less aggressive, i.e., it gets closer to $\tilde{\pi} = 1$ as ρ_w increases.

Why is $\tilde{\mu} < 1$ for all $\phi \neq 1$? Roughly, when $\phi < 1$, the main factor that dominates the policy choice is the banks' "disproportionately" high allocation on non-movers' consumption. This is because the bank perceives the consumption of the two types as gross substitutes and the return on cash that goes to movers' consumption is uncertain. Then $\tilde{\mu} < 1$, by transferring resources to movers, aligns banks' choices with what is ex-ante optimal.¹¹ On the other hand, when $\phi > 1$, banks' choice of deposits saved for movers is "too" volatile over time. Ex-ante, $\tilde{\mu} < 1$ then provides an appropriate compensation.¹²

Irrespective of the value of ϕ , the deviation of banks' choices from ex-ante optimality is primarily due to the money's rate of return uncertainty. As the unconditional distribution is preserved by assumption, an increase in the persistence of endowment shocks implies a reduced spread of its next period's conditional distribution. As a result, the banks choices are closer to what ex-ante optimality requires. This is evident from Figure 2 that shows that γ is closer to the average and less steep as ρ_w increases. This leads to the result shown in Figure 1(b): the policy is less aggressive

¹¹A planner constrained by the fact that movers must consume from current endowment will choose allocations such that ex-ante $E(u'(c_m)) = x E(u'(c_n))$. In a decentralized equilibrium however the bank's choice is governed by $E(u'(c_m) R_m) = x E(u'(c_n))$. Setting $\tilde{\mu} < 1$ aligns the decentralized marginal condition with that of the planner.

¹²For technical details of this argument the reader is referred to Bhattacharya and Singh (2007), who obtain similar results for i.i.d. shocks.

as ρ_w increases, i.e., it gets closer to $\tilde{\mu} = 1$.

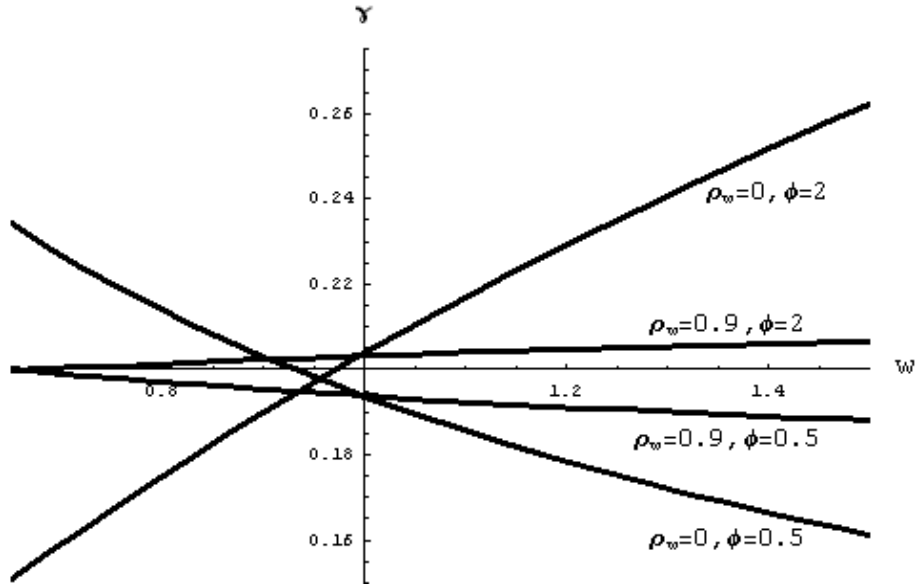


Figure 2: γ as a function of ρ_w under monetary targeting

Figure 3 presents the percentage gain in steady state welfare, expressed in terms of equivalent consumption, that is obtained under monetary targeting relative to

inflation targeting.

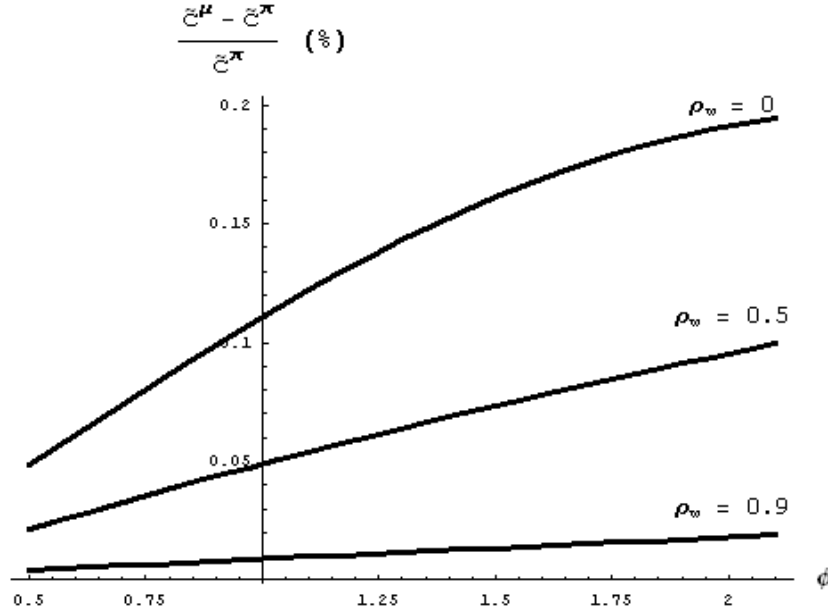


Figure 3: % change in \tilde{c}^i from following $\tilde{\mu}$ over $\tilde{\pi}$

Proposition 3 showed that the welfare gap between monetary and inflation targeting is decreasing in ρ_w ; the same holds for all ϕ . Figure 3 also shows that the welfare gap is increasing in ϕ for any given ρ_w . Intuitively, a relatively higher income volatility under inflation targeting hurts more as ϕ increases, thus making monetary targeting even more desirable.

In the next section, we study uncertainty regarding the return to the storage technology. In that case, as we demonstrate below, the higher the persistence of the storage return shocks, the *larger* is the welfare gain under the welfare-superior regime.

4 Uncertain return on storage

In this section, we shut off the endowment uncertainty and instead allow the return on storage to be uncertain. Specifically, we assume that each unit of the consumption good stored at date $t - 1$ yields x_t units at the start of date t . Further x_t is assumed

to follow an AR(1) process:

$$x_t - x^e = \rho_x (x_{t-1} - x^e) + \varepsilon_t; \quad \text{and } \varepsilon_t \sim \text{i.i.d.}$$

where it is assumed that $x^e > 1$; $\varepsilon_t \in [\underline{\varepsilon}, \bar{\varepsilon}]$ with $E_{t-1} \{\varepsilon_t\} = 0$ and represents shocks to the real interest rate. The above process implies that x_t is distributed over support $\left[x^e + \frac{\underline{\varepsilon}}{1-\rho_x}, x^e + \frac{\bar{\varepsilon}}{1-\rho_x}\right]$ and its unconditional mean and variance are given by $x^e > 1$ and $\sigma_x^2 \equiv \frac{\sigma_\varepsilon^2}{1-\rho_x^2}$, respectively. Conditionally, however, $E\{x_t|x_{t-1}\} = (1-\rho_x)x^e + \rho_x x_{t-1}$. It is further assumed that $E\{x_t|x_{t-1}\} > 1$ for all x_{t-1} , which requires that $x^e + \frac{\rho_x}{1-\rho_x}\underline{\varepsilon} > 1$ or $\underline{\varepsilon} > -\frac{1-\rho_x}{\rho_x}(x^e - 1)$. This ensures that a strictly positive amount of storage is held by banks at all times.

Our goal is to repeat the previous set of exercises, i.e., investigate how optimal monetary policy responds to real interest rate shocks, and in particular, on the persistence (ρ_x) of such shocks. Since the structure of the results is roughly similar to those presented above for endowment shocks, we will necessarily be more brief in their presentation below.

In the case of log utility, $\gamma_t = \alpha$ for all t ; in particular, γ is independent of x . Under monetary targeting, since $m_t = \alpha(w + \tau_t)$ and $\tau_t = \frac{M_t - M_{t-1}}{p_t} = m_t \left(1 - \frac{1}{\mu}\right)$, it follows that $m_t = \alpha w / \left(1 - \alpha \left(1 - \frac{1}{\mu}\right)\right)$ for all t . Further,

$$R_{mt} = \frac{p_t}{p_{t+1}} = \frac{1}{\mu} \frac{m_{t+1}}{m_t} = \frac{1}{\mu}.$$

Under inflation targeting, $R_{mt} = \frac{1}{\pi}$ for all t . Further, $m_t = \alpha(w + \tau_t)$ and $\tau_t = \frac{M_t - M_{t-1}}{p_t} = m_t - \frac{1}{\pi} m_{t-1}$. In turn, $m_t = -\frac{\alpha}{1-\alpha} \frac{1}{\pi} m_{t-1} + \frac{\alpha}{1-\alpha} w$. In a stationary equilibrium, $m_t = \alpha w / \left(1 - \alpha \left(1 - \frac{1}{\pi}\right)\right)$ for all t . Thus, in either case the indirect utility is given by

$$W^z = -\alpha \ln z + (1-\alpha) E_t \{\ln x_{t+1}|x_t\} + \ln w - \ln \left(1 + \alpha \left(1 - \frac{1}{z}\right)\right) \quad \text{for } z = \mu, \pi. \quad (22)$$

Proposition 4 *Irrespective of the specification of shocks to storage returns, monetary and inflation targeting are equivalent under logarithmic utility. The optimal rule requires fixing the money supply or equivalently the price level.*

Under log preferences, banks always spend α fraction of deposits to acquire cash reserves. For a fixed money supply, the net-of-transfers income is also simply the endowment – in this case, the banks keep aside a fraction α of the endowment to purchase cash. Prices are constant period after period and the rate of return on money is unity. The same equilibrium obtains if prices were instead fixed. But, why is a constant money supply optimal? To answer this, compare (22) with (16). The terms containing μ are independent of current and/or future return on storage. Once again, optimal monetary policy ignores the intergenerational margin and $\tilde{\mu} = 1$ obtains an efficient intragenerational margin.

Does this result continue to hold in the more general CRRA form of utility? We now show that the equivalence of the two instruments described in Proposition 4 breaks down when the storage shocks are persistent, i.e., $\rho_x > 0$. Under inflation targeting, the rate of return on money is fixed, i.e., $R_{mt} = \pi^{-1}$ for all t . The equilibrium γ_t [denoted $\gamma(\pi, x_t)$] is readily obtained from (6) as

$$\gamma(\pi, x_t) = \frac{\alpha}{\alpha + (1 - \alpha) \pi^{\frac{1-\phi}{\phi}} \left[E_t \left\{ x_{t+1}^{\phi-1} | x_t \right\} \right]^{-\frac{1}{\phi}}} \quad (23)$$

However, under monetary targeting, uncertain storage returns also contribute to return on money uncertainty. In this case, $\gamma(\mu, x_t)$ is obtained from (6) as

$$\gamma(\mu, x_t) = \frac{\alpha}{\alpha + (1 - \alpha) \left[\frac{\mu \gamma(\mu, x_t)}{1 - \gamma(\mu, x_t) \left(1 - \frac{1}{\mu}\right)} \right]^{-\frac{\phi-1}{\phi}} \left[E_t \left\{ x_{t+1}^{\phi-1} \left(\frac{1 - \gamma(\mu, x_{t+1}) \left(1 - \frac{1}{\mu}\right)}{\gamma(\mu, x_{t+1})} \right)^{\phi-1} \middle| x_t \right\} \right]^{-\frac{1}{\phi}}} \quad (24)$$

As is evident, the current γ does depend on the return to storage in the following period. If x is persistent, the distribution of x_{t+1} is contingent on x_t and in turn γ_t is a function of x_t ; otherwise not (as the following Proposition makes clear).

Proposition 5 *For CRRA utility, under i.i.d. shocks to storage returns ($\rho_x = 0$), monetary targeting and inflation targeting are equivalent. In either case, the best rule*

involves fixing the money supply or equivalently the price level. If $\rho_x > 0$ holds, the equivalence breaks down.

When banks allocate their deposits into cash and storage, the return on either of them is not known. While the return on storage is unknown by assumption, the return on money depends on the future price level which in turn depends on next period's allocations. When the return to storage is i.i.d., its current realization does not help predict its future values. Then γ_t is independent of x_t . As all other exogenous variables, including the money growth rate, are constant, γ is time-invariant, and so are the transfers and deposits at the bank. Once again, prices grow at the money growth rate. The rate of return on money and real balances are constant, which also means that movers' consumption is constant over time.

Why is $\tilde{\mu} = 1$ then? Notice that in the decentralized equilibrium movers directly consume a constant fraction of endowment, whereas the non-movers consume only stored goods with uncertain returns. From a planner's perspective a unit of endowment that goes to movers obtains $u'(c_m)$ whereas a unit reserved for non-movers obtains $E_t \{x_{t+1} u'(c_{nt})\}$. Equating the two and then comparing with (4) obtains $\tilde{\mu} = 1$.

On the other hand, when shocks are persistent x_t does help in predicting x_{t+1} and then γ_t does depend on x_t . A constant money supply does not lead to a stationary prices anymore, and the equivalence breaks down. What optimal money growth/inflation rates obtain for $\rho_x > 0$ (and $\phi \neq 1$)? Which targeting regime is superior for different values of ϕ ? As before, we resort to numerical simulations. Our choice of parametric specification is as follows. We fix $\alpha = 0.2$ and assume that the long-run distribution of x is log-normal. For the AR(1) specification, we assume

$$\ln x = \rho_x \ln x_{t-1} + \varepsilon_t,$$

where $\varepsilon \sim N(\mu(1 - \rho_x), \sigma_x^2(1 - \rho_x^2))$; $\rho_x = 0$ obviously nests the i.i.d. specification. Below we present results for $\rho_x = 0.5, 0.8$, and 0.85 . We compare optimal money

growth and inflation rates under the two policies, along with their respective welfare levels, for $\phi \in [0.5, 2.1]$.

We first rewrite the expression for period t welfare as (see (21))

$$W_t \equiv \frac{(1 - \alpha)^\phi (w + \tau_t)^{1-\phi} (1 - \gamma_t)^{-\phi} E_t \left(x_{t+1}^{1-\phi} | x_t \right) - 1}{1 - \phi} = \left(\frac{\alpha}{\gamma_t} \right)^\phi \frac{(w + \tau_t)^{1-\phi} E_t (R_{mt}) - 1}{1 - \phi}. \quad (25)$$

Under inflation targeting, it is clear from (23) that γ_t depends on the current realization of x_t . As γ varies stochastically over time, so will the net-of-transfers income that, following (13), is governed by

$$(\bar{w} + \tau_{t+1}) = -\frac{\gamma(\pi, x_t)}{(1 - \gamma(\pi, x_t)) \pi} (\bar{w} + \tau_t) + \frac{\bar{w}}{1 - \gamma(\pi, x_t)}. \quad (26)$$

With (23) and (26) it is straightforward to numerically simulate (25) over a large number of time periods. An average then obtains the ex-ante utility for any given π . As under endowment uncertainty, $\tilde{\pi}$ is obtained as $\arg \max (W^\pi)$. Under monetary targeting, the main step entails computing the fixed point of γ as a function of x . As under endowment shocks, we guess an initial function $\gamma = \alpha$, and numerically iterate on (24) to convergence. This is done for a fixed μ . Once the γ function is obtained, evaluating (25) is straightforward. By repeating this exercise for different values of μ , we easily obtain $\tilde{\mu}$ and \tilde{W}^μ .

Figure 4 below shows that, just as in the case of endowment shocks, $\tilde{\pi} \geq 1$ for all

ϕ . Interestingly, $\tilde{\mu} \geq 1$ for all $\phi \geq 1$ obtains.

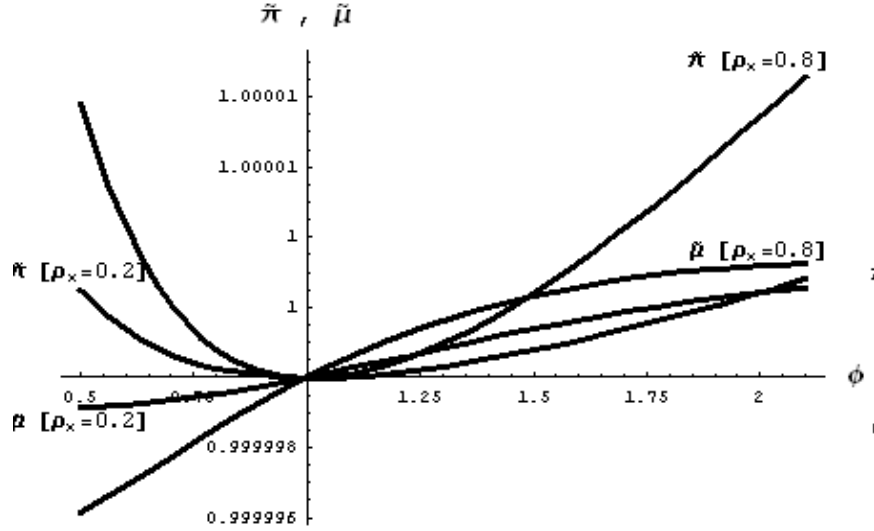


Figure 4: Optimal μ and π against ϕ (storage shocks)

For the log case, we know from Proposition 4 that $\tilde{\mu} = \tilde{\pi} = 1$ for all ρ_x . For $\rho_x > 0$ and $\phi \neq 1$, as the discussion following Proposition 5 clarifies, γ_t fluctuates and so does τ_t . Then, as under endowment shocks, a higher π helps reduce the income volatility as apparent from (26), and therefore the policy intervention is inflationary. Why is $\tilde{\mu} \geq 1$ for $\phi \geq 1$? Intuitively, the uncertainty of storage returns hits the non-movers' consumption directly; its impact on the portfolio choice γ , based on future expectations, is only indirect and relatively minor. Thus, non-movers' consumption relative to movers' consumption is more volatile. An optimal policy tends to transfer income to non-movers when $\phi > 1$, i.e., when the two consumptions are gross complements. This is achieved by $\tilde{\mu} > 1$. The opposite is the case when $\phi < 1$.

Figure 4 also shows that, in sharp contrast to the policy responses under endowment shocks, the optimal money growth and inflation rate responses amplify with an increase in the persistence of storage shocks. Intuitively, as persistence increases, current realizations of storage returns are closer to its next period's predicted value (see the discussion following Proposition 5). As a result, banks' choices respond more

to the current shocks. The optimal policy responses then have to be more aggressive.

Finally, Figure 5 below shows that $\tilde{W}^\mu \geq \tilde{W}^\pi$ as $\phi \geq 1$.

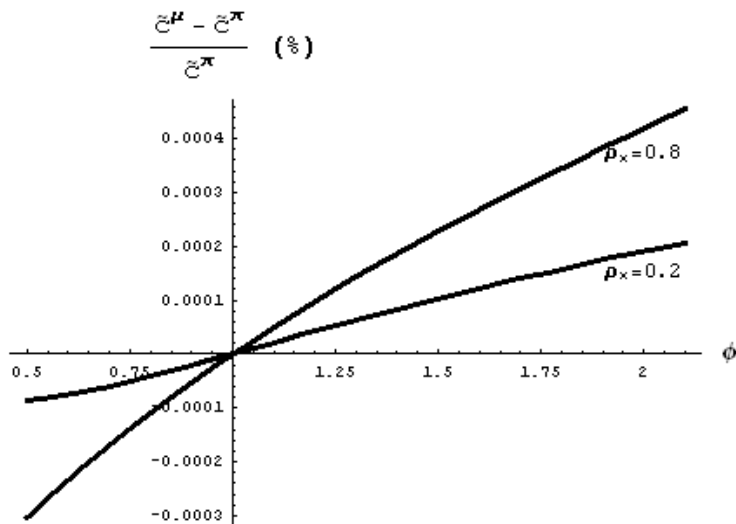


Figure 5: % change in \tilde{c}^i from following $\tilde{\mu}$ over $\tilde{\pi}$

A rough intuition for this result is as follows. Relative to monetary targeting, a fixed inflation target ensures a better intratemporal efficiency but generates a higher intertemporal income volatility. For the log case none of these effects are present. When the elasticity of substitution is low relative to the log case (i.e., $\phi > 1$), the welfare cost of a higher income volatility exceeds its benefits in terms intratemporal efficiency. The opposite is the case when $\phi < 1$.

5 Conclusion

In recent years, the random relocation model has become an important vehicle for research in monetary theory in the overlapping-generations tradition. Most work using this model has been done in a deterministic environment; in the few instances where shocks are allowed, they are restricted to be i.i.d. in nature. In this paper, we extend the scope of use of such models to the case of persistent shocks. We use

the structure to study the tug-o-war for supremacy between inflation targeting and monetary targeting, a classic yet timely topic in monetary economics. It is useful to note that absent shocks, there would be no difference between these instruments in our model.

We study shocks to the endowment as well as shocks to the real interest rate. We are able to obtain clean analytical results (with accompanying intuition) for the case of logarithmic utility. The analysis plays off two margins – the “intratemporal efficiency” or “intragenerational efficiency” condition connecting marginal utilities of movers and non-movers at any date and the intergenerational margin that appears here because shocks hit generations asymmetrically. In contrast to standard versions of these models, the optimal policy in our setup pays attention to providing some amount of intergenerational insurance. The net result is a trade-off between intratemporal for intertemporal efficiency and this causes optimal monetary policy to deviate from whatever policy achieves intratemporal efficiency alone.

Irrespective of the nature of shocks, the optimal inflation target is always positive. Under monetary targeting, shocks to endowment require negative money growth rates, while under shocks to real interest rates it may be either positive or negative depending on the elasticity of consumption substitution. Also, monetary targeting welfare-dominates inflation-rate targeting but the gap between the two vanishes as the shock process approaches a random walk. In sharp contrast, for shocks to the real interest rate, we prove that monetary targeting and inflation targeting are welfare-equivalent only in the limit when the shocks become i.i.d.

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Appendix

A Proof of Lemma 1

Under inflation targeting, the money supply follows an AR(1) process

$$m_t = -\rho_m m_{t-1} + \frac{\alpha}{1-\alpha} w_t,$$

which (under a moving-average representation) can be iteratively written as:

$$m_t = \frac{\alpha}{1-\alpha} \sum_{s=0}^{\infty} (-\rho_m)^s w_{t-s}, \quad (27)$$

Next, w is governed by

$$w_t = \rho_w w_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}$$

Let $E\{\varepsilon_t\} = w^e(1-\rho_w)$ and $\sigma_\varepsilon^2 = \sigma_w^2(1-\rho_w^2)$ so that the unconditional mean and variance are identical under the two specifications. Under a moving-average representation $w_t = \sum_{s=0}^{\infty} (\rho_w)^s \varepsilon_{t-s}$, which after substituting in (27) yields

$$m_t = \frac{\alpha}{1-\alpha} \left[\begin{aligned} &\sum_{s=0}^{\infty} \rho_w^s \varepsilon_{t-s} - \rho_m \sum_{s=1}^{\infty} \rho_w^{s-1} \varepsilon_{t-s} \\ &+ \rho_m^2 \sum_{s=2}^{\infty} \rho_w^{s-2} \varepsilon_{t-s} - \rho_m^3 \sum_{s=3}^{\infty} \rho_w^{s-3} \varepsilon_{t-s} \\ &+ \dots \end{aligned} \right].$$

Combining the time-indexed shock terms yields

$$m_t = \frac{\alpha}{1-\alpha} \left[\begin{aligned} &\varepsilon_t + (\rho_w - \rho_m) \varepsilon_{t-1} + (\rho_w^2 - \rho_m \rho_w + \rho_m^2) \varepsilon_{t-2} + \\ &(\rho_w^3 - \rho_m \rho_w^2 + \rho_m^2 \rho_w - \rho_m^3) \varepsilon_{t-3} + \\ &(\rho_w^4 - \rho_m \rho_w^3 + \rho_m^2 \rho_w^2 - \rho_m^3 \rho_w + \rho_m^4) \varepsilon_{t-4} + \dots \end{aligned} \right]. \quad (28)$$

To compute σ_m^2 , use equation (28) to obtain (using $\sigma_\varepsilon^2 = \sigma_w^2(1-\rho_w^2)$)

$$\sigma_m^2 = \frac{\alpha^2}{(1-\alpha)^2} \sigma_w^2 (1-\rho_w^2) \left[\begin{aligned} &1 + \rho_w^2 \left(1 - \frac{\rho_m}{\rho_w}\right)^2 + \rho_w^4 \left(1 - \frac{\rho_m}{\rho_w} + \left(\frac{\rho_m}{\rho_w}\right)^2\right)^2 \\ &+ \rho_w^6 \left(1 - \frac{\rho_m}{\rho_w} + \left(\frac{\rho_m}{\rho_w}\right)^2 - \left(\frac{\rho_m}{\rho_w}\right)^3\right)^2 \\ &+ \rho_w^8 \left(1 - \frac{\rho_m}{\rho_w} + \left(\frac{\rho_m}{\rho_w}\right)^2 - \left(\frac{\rho_m}{\rho_w}\right)^3 + \left(\frac{\rho_m}{\rho_w}\right)^4\right)^2 + \dots \end{aligned} \right].$$

(29)

Notice that the expression with the square brackets in the RHS remains identical by interchanging ρ_w with ρ_m . Therefore, without loss of generality, we assume that $\rho_w > \rho_m$ in what follows. To simplify notation, let $x \equiv \rho_w < 1$ and $y \equiv \frac{\rho_m}{\rho_w} < 1$. Then, the above leads to

$$\sigma_m^2 = \frac{\alpha^2}{(1-\alpha)^2} \sigma_w^2 (1-\rho_w^2) \left[1 + \left(x \frac{1-y^2}{1+y} \right)^2 + \left(x^2 \frac{1+y^3}{1+y} \right)^2 + \left(x^3 \frac{1-y^4}{1+y} \right)^2 + \dots + \right],$$

After expanding the terms within square brackets, making use of summation formulae for infinite series,¹³ and substituting back $x = \rho_w$ and $y = \frac{\rho_m}{\rho_w}$ leads to

$$\sigma_m^2 = \frac{\alpha^2}{(1-\alpha)^2} (1-\rho_w^2) \sigma_w^2 \left[1 + \frac{1}{(\rho_w + \rho_m)^2} \left[\frac{\rho_w^4}{1-\rho_w^2} - 2 \frac{\rho_m^2 \rho_w^2}{1+\rho_w \rho_m} + \frac{\rho_m^4}{1-\rho_m^2} \right] \right], \quad (30)$$

which is the equation (18) in Lemma 1.

Now, we show that σ_m^2 is monotonically declining in ρ_w . To do so we show that $\frac{d\sigma_m^2}{d\rho_w}$ is negative for $\rho_w = 0$, $\rho_w \rightarrow 1$, and $\rho_w = \rho_m$, for any value of $\rho_m \in (0, 1)$. It directly follows from (30) that

$$\left. \frac{d\sigma_m^2}{d\rho_w} \right|_{\rho_w=0} = -2 \frac{\alpha^2}{(1-\alpha)^2} \sigma_w^2 \frac{\rho_m}{(1-\rho_m^2)} < 0.$$

¹³The terms within square brackets can be expanded and recollected as

$$\begin{aligned} & 1 + \left(x \frac{1-y^2}{1+y} \right)^2 + \left(x^2 \frac{1+y^3}{1+y} \right)^2 + \left(x^3 \frac{1-y^4}{1+y} \right)^2 + \dots + \\ &= 1 + x^2 \frac{1+y^4-2y^2}{(1+y)^2} + x^4 \frac{1+y^6+2y^3}{(1+y)^2} + x^6 \frac{1+y^8-2y^4}{(1+y)^2} + \dots \\ &= 1 + \frac{1}{(1+y)^2} \left[(x^2 + x^4 + x^6 + \dots) + (x^2 y^4 + x^4 y^6 + x^6 y^8 + \dots) \right. \\ &\quad \left. - 2(x^2 y^2 + x^4 y^3 + x^6 y^4 + \dots) \right] \\ &= 1 + \frac{1}{(1+y)^2} \left[\frac{x^2}{1-x^2} + \frac{x^2 y^4}{1-x^2 y^2} - 2 \frac{x^2 y^2}{1+x^2 y} \right] \\ &= 1 + \frac{x^2}{(1+y)^2} \left[\frac{1}{1-x^2} + \frac{y^4}{1-x^2 y^2} - 2 \frac{y^2}{1+x^2 y} \right]. \end{aligned}$$

Similarly, after some algebra it can be shown that

$$\begin{aligned}
\frac{(1-\alpha)^2}{\alpha^2\sigma_w^2} \frac{d\sigma_m^2}{d\rho_w} \Big|_{\rho_w=1} &= -2 + \frac{4}{(1+\rho_m)^2} - \frac{2}{(1+\rho_m)^3} + 4 \frac{\rho_m^2}{(1+\rho_m)(1+\rho_m)^2} - 2 \frac{\rho_m^4}{(1-\rho_m^2)(1+\rho_m)^2} \\
&= -2 + \frac{2}{(1-\rho_m)(1+\rho_m)^3} [2(1-\rho_m^2) + (2\rho_m^2-1)(1-\rho_m) - \rho_m^4] \\
&= 2 \left(-1 + \frac{1}{(1-\rho_m)(1+\rho_m)^3} [1 + \rho_m - 2\rho_m^3 - \rho_m^4] \right).
\end{aligned}$$

Now,

$$1 + \rho_m - 2\rho_m^3 - \rho_m^4 < (1-\rho_m)(1+\rho_m)^3.$$

To see this expand the RHS and simplify to get

$$\rho_m < 2\rho_m,$$

which must hold for all $\rho_m > 0$. Hence, $\frac{d\sigma_m^2}{d\rho_w} \Big|_{\rho_w=1} < 0$. Finally,

$$\begin{aligned}
\frac{(1-\alpha)^2}{\alpha^2\sigma_w^2} \frac{d\sigma_m^2}{d\rho_w} \Big|_{\rho_w=\rho_m} &= -2\rho_m \left[1 + \frac{1}{4\rho_m^2} \left[-\frac{2\rho_m^4}{1+\rho_m^2} + \frac{2\rho_m^4}{1-\rho_m^2} \right] \right] \\
&\quad - \frac{2(1-\rho_m^2)}{8\rho_m^3} \left[\frac{2\rho_m^4}{1-\rho_m^2} - 2\frac{\rho_m^4}{1+\rho_m^2} \right] \\
&\quad + \frac{(1-\rho_m^2)}{4\rho_m^2} \left[\frac{4\rho_m^3 - 2\rho_m^5}{(1-\rho_m^2)^2} - \frac{4\rho_m^3 + 2\rho_m^5}{(1+\rho_m^2)^2} \right] \\
&= -\rho_m - \frac{1}{2} \frac{\rho_m^3}{(1-\rho_m^2)} - \frac{(1-\rho_m^2)}{4\rho_m^2} \frac{4\rho_m^3 + 2\rho_m^5}{(1+\rho_m^2)^2} < 0,
\end{aligned}$$

since all the terms in the last equality are negative.

B Proof of Lemma 2

Using (30) it is straightforward to obtain

$$\sigma_m^2 \begin{cases} = \frac{\alpha^2\sigma_w^2}{(1-\alpha)^2 - \alpha^2/\pi^2}, & \text{if } \rho_w = 0 \\ = \frac{\alpha^2\sigma_w^2}{(1-\alpha)^2 + \alpha^2/\pi^2}, & \text{if } \rho_w = \rho_m \\ = \frac{\alpha^2\sigma_w^2}{(1-\alpha+\alpha/\pi)^2}, & \text{as } \rho_w \rightarrow 1 \end{cases}. \quad (31)$$

Next, using (31) and (34), it is easily shown that (for any $\tilde{\pi} > 0$)

$$\frac{\frac{\sigma_m^2}{(m^e)^2}}{\frac{\sigma_w^2}{(w^e)^2}} \left\{ \begin{array}{l} = \frac{((1-\alpha)\tilde{\pi}+\alpha)^2}{(1-\alpha)^2\tilde{\pi}^2-\alpha^2} > 1 \text{ for } \rho_w = 0 \\ = \frac{((1-\alpha)\tilde{\pi}+\alpha)^2}{(1-\alpha)\tilde{\pi}^2+\alpha^2} > 1 \text{ for all } \rho_w = \rho_m \in (0, 1) \\ = 1 \text{ as } \rho_w \rightarrow 1 \end{array} \right. .$$

C Proof of Proposition 2

Rewriting (19):

$$W^\pi = -\alpha \ln \pi - \ln \alpha + (1 - \alpha) \ln x + E(\ln m(\pi)), \quad (32)$$

where the distribution of m is governed by (13). We first take a second-order Taylor approximation of the last term around m^e :

$$E\{\ln m\} \simeq \ln m^e - \frac{1}{2} \frac{\sigma_m^2}{(m^e)^2}. \quad (33)$$

Recall that $m^e = \frac{\alpha}{1-\alpha} \frac{w^e}{1+\rho_m} = \frac{\alpha w^e}{1-\alpha+\alpha/\pi}$. Then,

$$\frac{\sigma_m^2}{(m^e)^2} = \frac{(1-\alpha)^2}{\alpha^2 (w^e)^2} \sigma_m^2 (1+\rho_m)^2. \quad (34)$$

We are interested in differentiating (32) w.r.t. π . As $\rho_m = \frac{\alpha}{1-\alpha} \frac{1}{\pi}$ and $\frac{d\rho_m}{d\pi} = -\frac{\alpha}{1-\alpha} \frac{1}{\pi^2}$, for future use, we first prove that $\frac{d}{d\rho_m} \sigma_m^2 (1+\rho_m)^2 > 0$. We show that this holds for $\rho_m \rightarrow 0$, $\rho_m = \rho_w \in (0, 1)$, and $\rho_m < 1$. First, from (30) it is clear that as $\rho_m \rightarrow 1$, $\sigma_m^2 \rightarrow +\infty$. Next, using (29), one can derive

$$\begin{aligned} & \frac{1}{\sigma_w^2} \frac{1}{1-\rho_w^2} \left(\frac{1-\alpha}{\alpha} \right)^2 \frac{d}{d\rho_m} [\sigma_m^2 (1+\rho_m)^2] \\ &= 2(1+\rho_m) \left[\begin{array}{l} 1 + (\rho_w - \rho_m)^2 + (\rho_w^2 - \rho_m \rho_w + \rho_m^2)^2 \\ + (\rho_w^3 - \rho_m \rho_w^2 + \rho_m^2 \rho_w - \rho_m^3)^2 \\ + (\rho_w^4 - \rho_m \rho_w^3 + \rho_m^2 \rho_w^2 - \rho_m^3 \rho_w + \rho_m^4)^2 + \dots \end{array} \right] \\ & \quad + (1+\rho_m)^2 \left[\begin{array}{l} 2(\rho_m - \rho_w) + 2(\rho_w^2 - \rho_m \rho_w + \rho_m^2)(2\rho_m - \rho_w) + \\ 2(\rho_m^3 - \rho_w \rho_m^2 + \rho_w^2 \rho_m - \rho_w^3)(3\rho_m^2 - 2\rho_m \rho_w + \rho_w^2) \\ + 2(\rho_w^4 - \rho_m \rho_w^3 + \rho_m^2 \rho_w^2 - \rho_m^3 \rho_w + \rho_m^4) * \\ (4\rho_m^3 - 3\rho_m^2 \rho_w + 2\rho_m \rho_w^2 - \rho_w^3) + \dots \end{array} \right]. \end{aligned}$$

As $\rho_m \rightarrow 0$, the terms within the first square bracket equals $\frac{1}{1-\rho_w^2}$ while that in the second equals $\frac{-2\rho_w}{1-\rho_w^2}$. Then the RHS reduces to $\frac{2\sigma_w^2}{1+\rho_w} > 0$. Finally, when $\rho_m = \rho_w$, all the terms above are either zero or positive.

Notice however from (31) that as $\rho_w \rightarrow 1$, $\sigma_m^2 (1 + \rho_m)^2 = \frac{\alpha^2}{(1-\alpha)^2} \sigma_w^2$, and therefore $\frac{d}{d\rho_m} \sigma_m^2 (1 + \rho_m)^2 = 0$ as $\rho_w \rightarrow 1$. Thus, while the result above is valid for all $\rho_w \in [0, 1)$, it is not valid in the limit.

Using (34) with (33) in (32), and differentiating with respect to π yields

$$\begin{aligned} \frac{dW^\pi}{d\pi} &= -\frac{\alpha}{\pi} + \frac{\frac{\alpha}{\pi^2}}{1 - \alpha + \frac{\alpha}{\pi}} - \frac{1}{2} \frac{(1 - \alpha)^2}{\alpha^2 (w^e)^2} \frac{d[\sigma_m^2 (1 + \rho_m)^2]}{d\rho_m} \underbrace{\frac{d\rho_m}{d\pi}}_{=-\frac{\alpha}{1-\alpha} \frac{1}{\pi^2}} \\ &= \frac{\alpha}{\pi} \left[\frac{1}{(1 - \alpha)\pi + \alpha} - 1 \right] + \frac{1}{2} \frac{(1 - \alpha)}{\alpha (w^e)^2} \frac{d[\sigma_m^2 (1 + \rho_m)^2]}{d\rho_m}. \end{aligned}$$

Thus, for all $\rho_w \in [0, 1)$,

$$\left. \frac{dW^\pi}{d\pi} \right|_{\pi=1} = \frac{1}{2} \frac{(1 - \alpha)}{\alpha (w^e)^2} \frac{d[\sigma_m^2 (1 + \rho_m)^2]}{d\rho_m} > 0,$$

which, assuming W^π is concave in π , implies $\tilde{\pi} > 1$. As $\rho_w \rightarrow 1$, $\left. \frac{dW^\pi}{d\pi} \right|_{\pi=1} = 0$. Thus $\lim_{\rho_w \rightarrow 1} \tilde{\pi} = 1$.

D Proof of Proposition 3

We start by computing the maximized value of stationary welfare under monetary targeting, denoted by \tilde{W}^μ . From (17), it can be checked that at $\mu = 1$,

$$\tilde{W}^\mu = (1 - \alpha) \ln x + E \{ \ln w \}.$$

Analogous to (33), we can write $E \{ \ln w \} \simeq \ln w^e - \frac{1}{2} \frac{\sigma_w^2}{(w^e)^2}$ and so

$$\tilde{W}^\mu = (1 - \alpha) \ln x + \ln w^e - \frac{1}{2} \frac{\sigma_w^2}{(w^e)^2}.$$

Denote by W^π , the maximized value of stationary welfare under inflation targeting. From (19), it follows that

$$\tilde{W}^\pi = -\alpha \ln \tilde{\pi} - \ln \alpha + (1 - \alpha) \ln x + E(\ln m(\tilde{\pi})),$$

where it is known from Proposition 2 that $\tilde{\pi} > 1$. Using (33) with $m^e = \frac{\alpha w^e}{1-\alpha+\alpha/\pi}$, note that

$$\tilde{W}^\mu - \tilde{W}^\pi = \underbrace{\alpha \ln \tilde{\pi} + \ln \left(1 - \alpha + \frac{\alpha}{\tilde{\pi}}\right)}_{>0} + \frac{1}{2} \left(\underbrace{\frac{\sigma_m^2}{(m^e)^2} - \frac{\sigma_w^2}{(w^e)^2}}_{>0} \right) > 0. \quad (35)$$

Differentiating the first two terms, $\alpha \ln \tilde{\pi} + \ln \left(1 - \alpha + \frac{\alpha}{\tilde{\pi}}\right)$, with respect to $\tilde{\pi}$ yields a turning point at $\tilde{\pi} = 1$ and $\tilde{\pi} = \infty$. Checking the second derivative reveals that the sign is positive at $\tilde{\pi} = 1$ implying that $\alpha \ln \tilde{\pi} + \ln \left(1 - \alpha + \frac{\alpha}{\tilde{\pi}}\right)$ is a global minimum at $\tilde{\pi} = 1$ and so the minimum value of $\alpha \ln \tilde{\pi} + \ln \left(1 - \alpha + \frac{\alpha}{\tilde{\pi}}\right) = 0$. From Proposition 2 $\tilde{\pi} > 1$ for all $\rho_w < 1$, and therefore, $\alpha \ln \tilde{\pi} + \ln \left(1 - \alpha + \frac{\alpha}{\tilde{\pi}}\right) > 0$ for all $\rho_w < 1$. That the last term in brackets is positive follows from Lemma 2.

E Proof of Proposition 5

First, consider a fixed inflation target π . Then $R_{mt} = \pi^{-1}$ for all t , and γ is given by from (6) as

$$\gamma = \frac{\alpha}{\alpha + (1 - \alpha) \pi^{\frac{1-\phi}{\phi}} \left[E_t \left\{ x_{t+1}^{1-\phi} | x_t \right\} \right]^{\frac{1}{\phi}}}.$$

Since x is i.i.d., γ only depends on π as can be seen from the above equation, i.e., γ is constant over time. This policy implies a constant money growth rate given by $\mu_t = \frac{M_{t+1}}{M_t} = \frac{m_{t+1}}{m_t} \pi = \pi$ for all t , since $m_t = \gamma w$ for all t . Thus, monetary and inflation targeting are equivalent with any $\mu = \pi$.

For notational convenience below, let $X \equiv \left[E_t \left\{ x_{t+1}^{1-\phi} \right\} \right]^{\frac{1}{\phi}}$. Using $w + \tau = \frac{w}{1-\gamma(1-\frac{1}{\mu})}$ along with the above expression for γ after replacing π with μ in (25), the ex-ante welfare can be written as

$$W(\mu) = \frac{(1-\alpha)^\phi}{1-\phi} X^\phi \left(\frac{w}{1-\gamma \left(1-\frac{1}{\mu}\right)} \right)^{1-\phi} \left(\underbrace{\frac{(1-\alpha) \mu^{\frac{1-\phi}{\phi}} X}{\alpha + (1-\alpha) \mu^{\frac{1-\phi}{\phi}} X}}_{1-\gamma} \right)^{-\phi} - \frac{1}{1-\phi}.$$

The first order condition for welfare maximizing money growth/inflation rate is ob-

tained by differentiating the above with respect to μ :

$$\begin{aligned}
\frac{dW(\mu)}{d\mu} &= (1-\phi) \left(\frac{w}{1-\gamma\left(1-\frac{1}{\mu}\right)} \right)^{-\phi} (1-\gamma)^{-\phi} \frac{w}{\left(1-\gamma\left(1-\frac{1}{\mu}\right)\right)^2} \frac{\gamma}{\mu^2} \\
&\quad - (1-\phi) \left(\frac{w}{1-\gamma\left(1-\frac{1}{\mu}\right)} \right)^{-\phi} (1-\gamma)^{-\phi} \frac{w}{\left(1-\gamma\left(1-\frac{1}{\mu}\right)\right)^2} \left(1-\frac{1}{\mu}\right) \frac{d\gamma}{d\mu} \\
&\quad - \phi \left(\frac{w}{1-\gamma\left(1-\frac{1}{\mu}\right)} \right)^{1-\phi} (1-\gamma)^{-\phi-1} \frac{\alpha}{\left(\alpha+(1-\alpha)\mu^{\frac{1-\phi}{\phi}}X\right)^2} \left(\frac{1-\phi}{\phi}\right) (1-\alpha) X \mu^{\frac{1-\phi}{\phi}-1}.
\end{aligned}$$

Evaluate the above at $\mu = 1$; the second term is zero. The rest can be simplified to (after again using $\gamma = \frac{\alpha}{\alpha+(1-\alpha)\mu^{\frac{1-\phi}{\phi}}X^{\frac{1}{\phi}}}$):

$$\begin{aligned}
\left. \frac{dW(\mu)}{d\mu} \right|_{\mu=1} &= (1-\phi) \left(\frac{w}{1-\gamma\left(1-\frac{1}{\mu}\right)} \right)^{1-\phi} (1-\gamma)^{-\phi} \left(\frac{1}{1-\gamma\left(1-\frac{1}{\mu}\right)} \frac{\gamma}{\mu^2} - \frac{\gamma}{\mu} \right) \\
&= 0.
\end{aligned}$$